Coherence detection in a spiking neuron via Hebbian learning

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Abstract

It is generally assumed that neurons communicate through temporal firing patterns. As a first step, we will study the learning of a layer of realistic neurons in the particular case where the relevant messages are formed by temporally correlated patterns, or *synfire patterns*. The model is a layer of Integrate-and-Fire (IF) neurons with synaptic current dynamics that adapts by minimizing a cost according to a gradient descent scheme. The cost we define leads to a rule similar to Spike-Time Dependent Hebbian Plasticity (STDHP). Moreover, our results show that the rule that we derive is biologically plausible and leads to the detection of the coherence in the input in an unsupervised way. An application to shape recognition is shown as an illustration.

Key words: Spiking Neural Networks, Hebb Rule, Spike Time Dependent Hebbian Plasticity, Gradient Descent

1 Description of the model

1.1 Coding scheme

We will represent (as in [Gerstner99]) the signal S_i at synapse i by the sum of Dirac pulses located at the spiking times t_i^k drawn from the lists of spikes Γ_i (see Fig. 1-left).

$$S_i = \sum_{k \in \Gamma_i} \delta(t - t_i^k) \quad (1)$$

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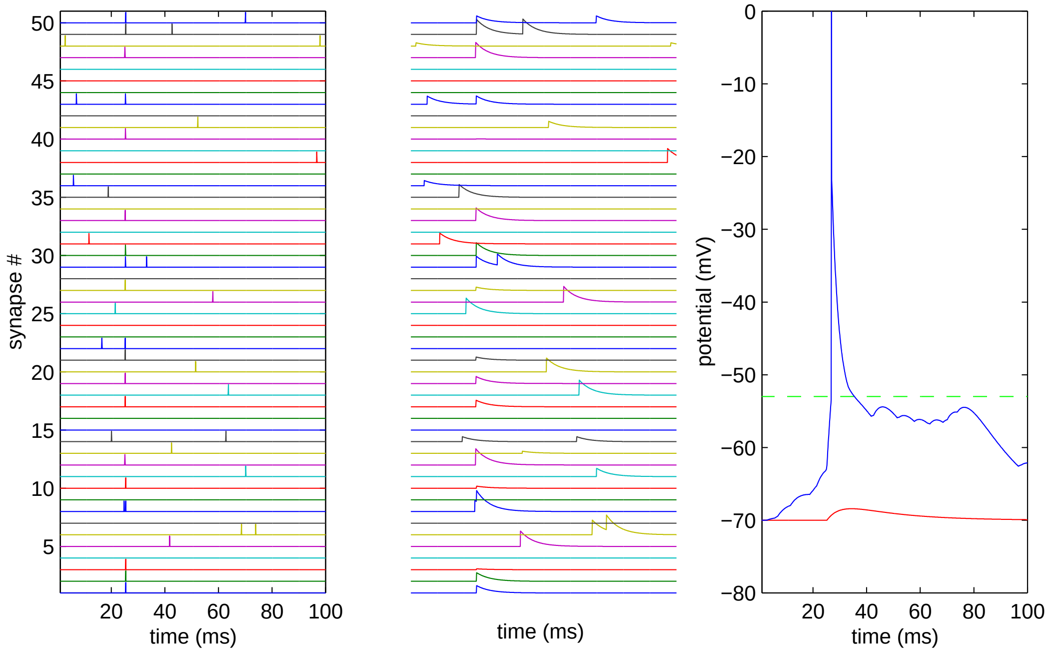

Fig. 1. Neuron Model : *(left)* Input spikes (with a synfire pattern at $t = 25ms$), are *(middle)* modulated in time and amplitude forming postsynaptic current pulses and are finally *(right)* integrated at the soma. When the potential (plain line) reaches the threshold (dotted line), a spike is emitted and the potential is decreased. A sample PSP (synapse 1) is also shown.

Synfire patterns are generated in analogy with the response of a retina to flashed binary images. The input of the synapses is characterized as the output of single-synapse IF neurons responding to a specific binary input.This response may be described as the sum of two random point processes with different time scales. At a narrow time scale, the input is the spontaneous activity, i.e. a background noise independent of time and synapses that may be described by a Poisson point process of rate $1/\tau_{noise}$. At a larger time scale, the synfire pattern activates a given subset $\mathcal{M}$ of synapses once per flash with a temporal correlation defined by its jitter τ_{jitter}. (see Fig. 1-left)

1.2 Integrate-and-Fire Layer

We will consider N_1 synapses (indexed by i) connected to a layer of N_2 neurons j. Those are generalized version of IF neurons with synaptic current dynamics, a one compartment model with no delay and the synapses have contacts characterized by their weight w_{ji}. The state variables are $N_1.N_2$ synaptic driving conductances g_{ji} and N_2 membrane potentials V_j. Incoming spikes trigger those conductances by opening the driving gates with time constant τ_g:

$$\frac{dg_{ji}}{dt} = -\frac{1}{\tau_g} g_{ji} + w_{ji}.S_i \tag{2}$$

and the potential V_j at the soma integrates with time constant τ_V the driving currents and the leaking current g_{leak} (with a potential $V_{rest} \approx -70\,mV$): (see Fig. 1-middle):

$$\tau_V \frac{dV_j}{dt} = g_{leak}.(V_{rest} - V_j) + (\textstyle\sum_{1 \leq i \leq N_2} g_{ji}) \tag{3}$$

When V_j reaches the threshold's potential $V_{threshold} \approx -54\,mV$ from below, the target neuron fires (see Fig. 1-right) and is shunted (V_j is set e.g. to $V_{reset} \approx -75\,mV$).

1.3 Reduced equations

We introduce reduced equations for this IF Layer to study its dynamical behavior and simplify its implementation. In fact, this reduction follows the concept of the Spike-Response Model (SRM) which was extensively studied in [Gerstner99,Kempter99]. It is similar to [Perrinet01] which aimed at reducing the STDHP equations to a set of first order equations.

$$\begin{cases} \frac{dc_i}{dt} = -\frac{1}{\tau_g} c_i + S_i \\ \tau_V \frac{dp_i}{dt} = -p_i + c_i \end{cases} \tag{4}$$

then, $V_j = V_{rest} + (\sum_{1 \leq i \leq N_1} w_{ji}.p_i)$ verifies the equation system (2, 3), with $g_{ji} = w_{ji}.c_i$.
To account for the threshold mechanism at a spiking time t_j^k, we may then add a resetting value to V_j by setting $\eta_j(t_j^k) = V_{reset} - V_{threshold}$ and then:

$$\tau_V \frac{\eta_j(t)}{dt} = -\eta_j(t) \tag{5}$$

So that finally, an equivalent version of the IF Layer consists of (4, 5) and :

$$V_j(t) = V_{rest} + (\textstyle\sum_{1 \leq i \leq N_1} w_{ji}.p_i(t)) + \eta_j(t) \tag{6}$$

This formulation depends only on the present state and not on the past values. It is therefore biologically more plausible and computationally cheaper.

Integrating these equations after emission of a presynaptic spike at t_i or a postsynaptic spike at t_j leads to :

$$c_i(t) = exp(-\frac{t - t_i}{\tau_g}) \tag{7}$$

$$p_i(t) = \frac{\tau_g}{\tau_V - \tau_g}(exp(-\frac{t - t_i}{\tau_g}) - exp(-\frac{t - t_i}{\tau_v})) \tag{8}$$

$$\eta(t) = (V_{reset} - V_{threshold})exp(-\frac{t - t_j}{\tau_V}) \tag{9}$$

Those equations (6, 7, 8 and 9) are the equivalent SRM version of our IF model. More precisely eq. 7 represents the PostSynaptic Current (PSC), see fig. 1-middle, and eq. 8 the PostSynaptic Potential (PSP), see fig. 1-right, which may be experimentally observed.

2 The learning mechanism

2.1 Definition of the cost function

Based on neurophysiological studies, we set the following principles :

(1) the learning is associated with a spiking response : the n^{th} learning step occurs at the n^{th} output firing time t_n,
(2) to discriminate between the different input patterns, the output voltage should be close to a winner-take-all configuration: the potential of the winning neuron (which we index $j = j_n$) should be above threshold whereas other neurons should be hyperpolarized,
(3) economy of the total synaptic efficacy and current use should be respected.

A possible cost function may therefore be the squared distance to the potentials of neurons at the firing time t_n added to the total sum of the squared weights:

$$2.E = \sum_{1 \le j \le N_2} (V_j - V_j^t)^2 + \alpha \sum_{1 \le j \le N_2} (\frac{\partial V_j}{\partial t})^2 + \beta \sum_{1 \le i \le N_1}^{1 \le j \le N_2} {w_{ji}}^2 \tag{10}$$

$$V_j^t = V_{rest} \qquad \text{for} \qquad j \neq j_n \tag{11}$$

$$V_{j_n}^t = V_{threshold} + \Delta V \tag{12}$$

Where α and β are scaling parameters and we set $\Delta V \approx 5\,mV$.

2.2 Gradient descent

It follows from equations 10 and 6 :

$$\begin{aligned}\frac{\partial E}{\partial w_{ji}} &= (V_j - V_j^t)\frac{\partial V_j}{\partial w_{ji}} + \alpha\frac{\partial V_j}{\partial t}\frac{\partial}{\partial w_{ji}}\frac{\partial V_j}{\partial t} + \beta w_{ji} \\ &= (V_j - V_j^t).p_i + \alpha\frac{\partial V_j}{\partial t}.\frac{dp_i}{dt} + \beta w_{ji}\end{aligned}$$

We may therefore formulate the gradient descent algorithm in our model as :

$$\begin{aligned}w_{ji}^{n+1} &= w_{ji}^n - \gamma_n.\frac{\partial E}{\partial w_{ji}} \\ &= (1-\beta_n)w_{ji}^n + \gamma_n(V_j^t - V_j).\frac{\partial V_j}{\partial w_{ji}} + \alpha_n.\frac{\partial V_j}{\partial t}.\frac{\partial \dot{V}_j}{\partial w_{ji}}\end{aligned}$$

with learning factors γ_n, α_n and β_n which satisfies (e.g. for γ_n) $\sum_{n=1\dots\infty}\gamma_n \to \infty$ and $\sum_{n=1\dots\infty}{\gamma_n}^2 < \infty$.

Finally, writing $\dot{p}_i = \frac{dp_i}{dt}$,

$$w_{ji}^{n+1} = (1-\gamma_n)w_{ji}^n + \alpha_n(V_j^t - V_j).p_i + \beta_n(\sum w_{ji}.\dot{p}_i)\dot{p}_i \qquad (13)$$

2.3 Spike-time Dependent Plasticity

A closer look at equation 13 shows that a direction in the change of w_{ji} is proportional to $(V_j^{target} - V_j).p_i$. This is the hebbian part of the rule : when a neuron j fires after the firing of synapse j, there is a mechanism that strengthen the connection. The strengthening depends therefore on the relative time of the pre- and post-synaptic spikes (see fig. 2.3) as is observed in biological systems [Bi98].

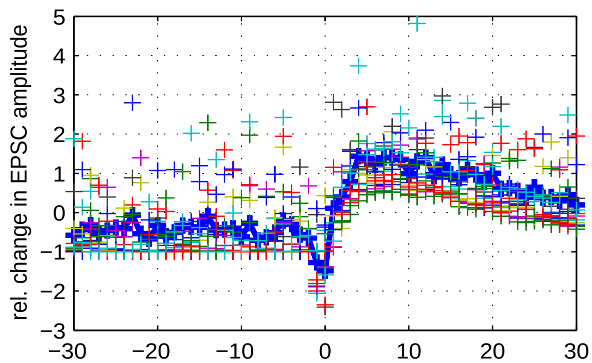

Fig. 2. STDHP : Relative weight change versus time difference between input and output spikes in a single neuron modelized with our model replicating the experimantal conditions of [Bi98].

3 Numerical results

We implemented this model using discrete versions of the differential equations (forward Euler method) on a MATLAB system.

3.1 Response to synfire patterns

To achieve this experiment we presented synfire patterns to the layer. The weights were set at random so that the network could fire to all the inputs. The patterns were presented at random times that were sufficiently distant. This unsupervised learning converges quickly, and as may be observed in neuromuscular connectivity, the synapses tend to sparsify and the neurons tend to respond to only one input (see Figure 3).

3.2 Response to oriented bars

The next experiment consisted in applying those results to a basic retina which input consists of centered rotated lines. A fixed analogical contrast layer (ON and OFF radial cells) sends then spikes to the learning layer that adapts with the rule we presented. We observe unsupervised emergence of V1-like receptors fields sensitive to the orientation (see Figure 4). Further experiments with lateral interactions and accounting for dendritic delay show even more realistic filters and column architecture.

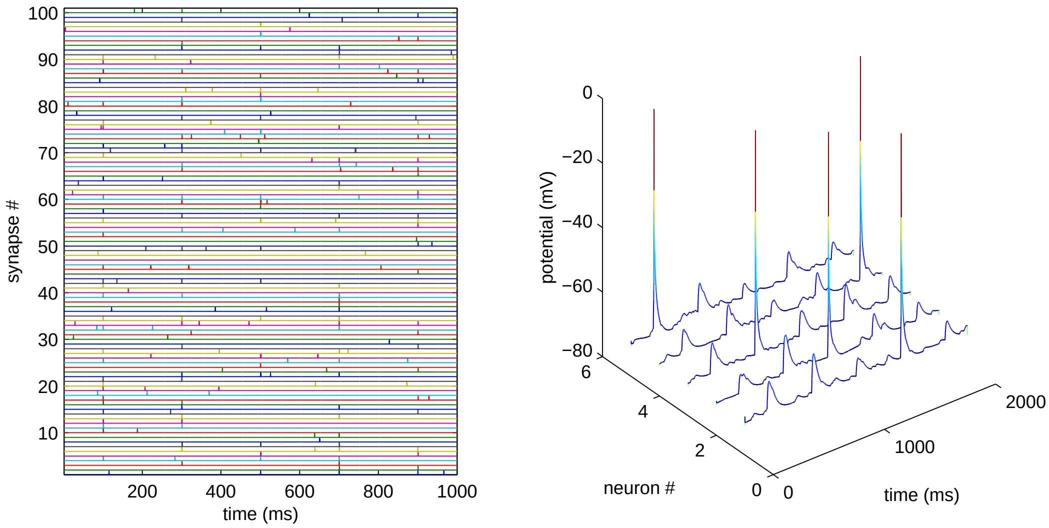

Fig. 3. Coherence detection: *(left)* different input patterns (firing times $t = 100ms, 300ms, 500ms, 700ms, 900ms$) are *(right)* learnt by the system : only one neuron per input fires (100 learning steps)

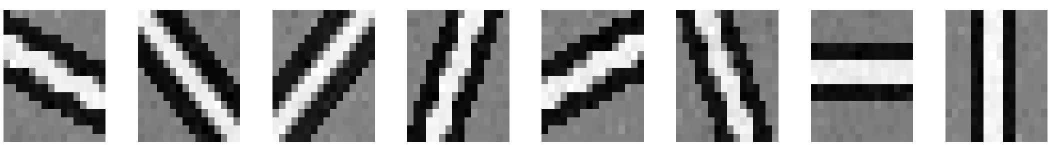

Fig. 4. Oriented bars detection: after learning, the weights show sensitivity to orientation (black:OFF; white:ON; gray:neutral)

Conclusion

We have presented an original gradient descent method to find a learning rule for a layer of spiking neurons. The simplicity of the rule gives a new insight into the comprehension of the mechanism behind the observed STDHP. Further work is done for the detection of asynchronous patterns.
However, this study should be extended to more realistic spike trains (e.g. bursts), account for more complex behavior (e.g. facilitation and depression) and may be extended to population of neurons and recurrent systems.

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Online simulations

http://laurent.perrinet.free.fr/app/app.html

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