The role of prediction in detecting motion
Probabilistic models of the low-level visual system

Laurent U. Perrinet

INCM, CNRS / Université de la Méditerranée
31, ch. Joseph Aiguier, 13402 Marseille Cedex 20, France

Friday, 17 December 2010
Solutions to the aperture problem
Solutions to the aperture problem
Solutions to the aperture problem

Figure 2 Evolution of direction tuning.

Figure 3 Visual tracking of an orientated bar.
Outline: The role of prediction in detecting motion

Probabilistic representations of motion
  Local measurement of motion
  Spatio-temporal integration of local measurements
  Toward global motion measurement

Our hypothesis: motion-based prediction
  Motion-based predictive coding
  Markov Chain
  Particle filtering (Sequential Monte-Carlo)

Prediction & the emergence of low-level computations
  Prediction is sufficient to solve the line’s aperture problem
  Texture-independent motion grabbers
  Tracking behaviour
  Interaction of different motions
Outline: The role of prediction in detecting motion

Probabilistic representations of motion
  Local measurement of motion
  Spatio-temporal integration of local measurements
  Toward global motion measurement

Our hypothesis: motion-based prediction
  Motion-based predictive coding
  Markov Chain
  Particle filtering (Sequential Monte-Carlo)

Prediction & the emergence of low-level computations
  Prediction is sufficient to solve the line’s aperture problem
  Texture-independent motion grabbers
  Tracking behaviour
  Interaction of different motions
Local measurement of motion

A

quadrature pair

$G_e$ $G_0$

$\wedge^2$

$\wedge^2$

$
\begin{array}{c}
\times \\
A \rightarrow B' \rightarrow A' \\
\end{array}$

motion energy

B

$\rightarrow f_1(x)$

$\rightarrow h_1(t)$

$\rightarrow h_2(t)$

$\rightarrow h_2(t)$

$\rightarrow h_1(t)$

Reichardt Detector

$\rightarrow +$

$\rightarrow -$
Local measurement of motion
Local measurement of motion
Spatio-temporal integration of local measurements

**Fig. 1. Model architecture.** Left: Neural model. Motion and form pathway are modelled simulating area V1, V2, and MT. Feedforward interactions are indicated using light gray arrows, modulatory interactions using dark gray arrows. Right: Cell types. V1 cells simply integrate over a small area, V2 RFs have the shape of a bipole. End-stop positions and T-junctions are indicated by the multiplicative combination of the depicted subfields. In MT, both Motion and Motion+Stereo cells are integrating their input (influence of V2 Form on MT Motion and of V1 Form on V1 Motion not depicted here), whereas the Motion contrast cells have a different velocity tuning in the center and the surround.
Spatio-temporal integration of local measurements
Spatio-temporal integration of local measurements
Outline: The role of prediction in detecting motion

Probabilistic representations of motion
  Local measurement of motion
  Spatio-temporal integration of local measurements
  Toward global motion measurement

Our hypothesis: motion-based prediction
  Motion-based predictive coding
  Markov Chain
  Particle filtering (Sequential Monte-Carlo)

Prediction & the emergence of low-level computations
  Prediction is sufficient to solve the line’s aperture problem
  Texture-independent motion grabbers
  Tracking behaviour
  Interaction of different motions
Motion-based predictive coding

\[ V(t-dt) \]

\[ x(t-dt) \]
Motion-based predictive coding

\[ V(t-dt) \]

\[ x(t-dt) \]

\[ x(t-dt) + V(t-dt) \cdot dt \]

\[ V(t-dt) \]
Motion-based predictive coding

\[ x(t) = x(t-dt) + V(t-dt) \cdot dt \]
Markov Chain

\[ p(x_{t-dt}, V_{t-dt} | I_{0:t-dt}) \]

\[ p(x_t, V_t | I_{0:t-dt}) \]

\[ x(t-dt) + V(t-dt) \cdot dt \]

\[ V(t) \]

\[ V(t-dt) \]

\[ x(t-dt) + V(t-dt) \cdot dt \]
Markov Chain

\[ p(x_t, V_t | I_{0:t-\Delta t}) \]

\[ p(x_t, V_t | I_{0:t}) \]

\[ p(I_t | x_t, V_t) \]
Markov Chain

\[ p(x_{t-dt}, V_{t-dt}|I_{0:t-dt}) \]
\[ p(x_t, V_t|I_{0:t}) \]
\[ p(l_t|X_t, V_t) \]
Particle filtering (Sequential Monte-Carlo)

Figure 5. One time-step in the Condensation algorithm: Each of the three steps—drift-diffuse-measure—of the probabilistic propagation process of Fig. 2 is represented by steps in the Condensation algorithm.
Outline: The role of prediction in detecting motion

Probabilistic representations of motion
  Local measurement of motion
  Spatio-temporal integration of local measurements
  Toward global motion measurement

Our hypothesis: motion-based prediction
  Motion-based predictive coding
  Markov Chain
  Particle filtering (Sequential Monte-Carlo)

Prediction & the emergence of low-level computations
  Prediction is sufficient to solve the line’s aperture problem
  Texture-independent motion grabbers
  Tracking behaviour
  Interaction of different motions
Prediction is sufficient to solve the line’s aperture problem
Prediction is sufficient to solve the line’s aperture problem
Texture-independent motion grabbers
Prediction & the emergence of low-level computations
Prediction & the emergence of low-level computations

\[
x(t) = x(t-dt) + V(t-dt) \cdot dt
\]

- \( V(t-dt) \)
- \( x(t-dt) \)
- \( V(t) \)
- \( \text{internal noise} \)
Prediction & the emergence of low-level computations
Prediction & the emergence of low-level computations
Prediction & the emergence of low-level computations
Prediction & the emergence of low-level computations
Interaction of different motions
\[ x(t) = x(t-dt) + V(t-dt) \cdot dt \]
For more information:

http://www.incm.cnrs-mrs.fr/LaurentPerrinet
Outline: The role of prediction in detecting motion

Appendix

References
For Further Reading
Bayes & motion
Condensation algorithm
Particle filtering
Homeostasis: histogram equalization
Spatio-temporal integration
Generative model
Spontaneous state of the dynamical system
P.-Y. Burgi, A. L. Yuille, and N. M. Grzywacz.
Probabilistic motion estimation based on temporal coherence.

M. Isard and A. Blake.
Condensation – conditional density propagation for visual tracking.

P. Seriès, S. Georges, J. Lorenceau, and Y. Frégnac.
Orientation dependent modulation of apparent speed: a model based on the dynamics of feed-forward and horizontal connectivity in V1 cortex.

S. Watamaniuk, S. McKee, and N. Grzywacz.
Detecting a trajectory embedded in random-direction motion noise.
For Further Reading

Laurent U. Perrinet.
*Topics in Dynamical Neural Networks: From Large Scale Neural Networks to Motor Control and Vision*, volume 142 of *The European Physical Journal (Special Topics)*, chapter Dynamical Neural Networks: modeling low-level vision at short latencies, pages 163–225.
URL http://www.incm.cnrs-mrs.fr/LaurentPerrinet/Publications/Perrinet06.

Laurent U. Perrinet and Guillaume S. Masson,
Modeling spatial integration in the ocular following response using a probabilistic framework.
*Journal of Physiology (Paris)*

Frédéric Barthélemy, Laurent U. Perrinet, Eric Castet and Guillaume S. Masson,
Dynamics of distributed 1D and 2D motion representations for short-latency ocular following.
Bayes & motion

\[ \mathbf{I}(\mathbf{x} + \mathbf{v}.dt, t + dt) = \mathbf{I}(\mathbf{x}, t) + \mathbf{v} \]
Bayes & motion

- \( \mathbf{I}(\vec{x} + \vec{v}.dt, t + dt) = \mathbf{I}(\vec{x}, t) + \nu \)
- A posteriori Probability
  - Bayes theorem: \( P(\vec{v}|\mathbf{I}) \propto P(\mathbf{I}|\vec{v}).P(\vec{v}) \)
  - Prior \( P(\vec{v}) = \frac{1}{\sqrt{2\pi\sigma_p}} e^{-\frac{\|\vec{v}\|^2}{2\sigma_p^2}} \)
  - Likelihood \( P(\mathbf{I}|\vec{v}) = \frac{1}{\sqrt{2\pi\sigma_m}} e^{-\frac{\mathcal{T}(\mathbf{I})^2}{2\sigma_m^2}} \)
  - with \( \mathcal{T}(\mathbf{I}) = \|\mathbf{I}(\vec{x}, t) - \mathbf{I}(\vec{x} - \vec{v}.dt, t - dt)\|^2 \) and \( \mathbf{I} = C.I_{100} \)
Bayes & motion

- \( \mathbf{I}(\vec{x} + \vec{v}.dt, t + dt) = \mathbf{I}(\vec{x}, t) + \nu \)

- A posteriori Probability
  - Bayes theorem: \( P(\vec{v}|\mathbf{I}) \propto P(\mathbf{I}|\vec{v}).P(\vec{v}) \)
  - Prior \( P(\vec{v}) = \frac{1}{\sqrt{2\pi}\sigma_p} e^{-\frac{||\vec{v}||^2}{2\sigma_p^2}} \)
  - Likelihood \( P(\mathbf{I}|\vec{v}) = \frac{1}{\sqrt{2\pi}\sigma_m} e^{-\frac{T(\mathbf{I})}{2\sigma_m^2}} \)
  - with \( T(\mathbf{I}) = ||\mathbf{I}(\vec{x}, t) - \mathbf{I}(\vec{x} - \vec{v}.dt, t - dt)||^2 \)
  - and \( \mathbf{I} = C \cdot \mathbf{I}_{100} \)

\[
P(\vec{v}|\mathbf{I}) \propto \exp\left(-\frac{C^2 \cdot T(\mathbf{I}_{100})^2}{2\sigma_m^2} - \frac{||\vec{v}||^2}{2\sigma_p^2}\right)
\]
Bayes & motion

- \( I(\vec{x} + \vec{v}.dt, t + dt) = I(\vec{x}, t) + \nu \)

- A posteriori Probability
  - Bayes theorem: \( P(\vec{v}|I) \propto P(I|\vec{v}).P(\vec{v}) \)
  - Prior \( P(\vec{v}) = \frac{1}{\sqrt{2\pi}\sigma_p} e^{-\frac{||\vec{v}||^2}{2\sigma^2_p}} \)
  - Likelihood \( P(I|\vec{v}) = \frac{1}{\sqrt{2\pi}\sigma_m} e^{-\frac{T(I)}{2\sigma^2_m}} \)
  - with \( T(I) = ||I(\vec{x}, t) - I(\vec{x} - \vec{v}.dt, t - dt)||^2 \)
  
  and \( I = C.I_{100} \),

\[
P(\vec{v}|I) \propto \exp\left(-\frac{C^2.T(I_{100})^2}{2\sigma^2_m} - \frac{||\vec{v}||^2}{2\sigma^2_p}\right)
\]

- Decision (eye’s acceleration gain): \( \gamma \propto \vec{v}^* \)
  
  \( \vec{v}^* = E(\vec{v}|I) = \int \vec{v}dP(\vec{v}|I) \)
Figure 5. One time-step in the Condensation algorithm: Each of the three steps—drift-diffuse-measure—of the probabilistic propagation process of Fig. 2 is represented by steps in the Condensation algorithm.
Particle filtering

\[(x_i, y_i)_{\text{t}} = (x_i, y_i)_{\text{t-1}} + \delta_{\text{t}} \cdot (u_i, v_i)_{\text{t-1}} + (\nu_x, \nu_y)_{\text{t}} \] (1)

\[(u_i, v_i)_{\text{t}} = (u_i, v_i)_{\text{t-1}} + \delta_{\text{t}} \cdot (\nu_u, \nu_v)_{\text{t}} \] (2)
Particle filtering

\[(x_i, y_i)_t = (x_i, y_i)_{t-\delta t} + \delta t (u_i, v_i)_{t-\delta t} + (\nu_x, \nu_y)\] (1)

\[(u_i, v_i)_t = (u_i, v_i)_{t-\delta t} + (\nu_u, \nu_v)\] (2)
Particle filtering

\[(x_i, y_i)_{t = (x_i, y_i)_{t - \delta t} + \delta t (u_i, v_i)}_{t - \delta t} + (\nu_x, \nu_y)} (1)\]

\[(u_i, v_i)_{t = (u_i, v_i)_{t - \delta t} + (\nu_u, \nu_v)} (2)\]
Figure 1. Probability distribution of contrasts, (a), in the fly environment from the measurements of Laughlin (1981). The contrast-response predicted by information theory is the cumulative probability map in (b). (c) is a comparison between the predicted response and that actually measured by Laughlin (1981) in the LMC.
Spatio-temporal integration
Spatio-temporal integration

How is probabilistic information pooled?

\[
\mathcal{N}(\vec{\nu}_n, C_n) = \frac{1}{\sqrt{\det(2\pi C_n)}} e^{-\frac{(\vec{\nu} - \vec{\nu}_n)^T C_n^{-1} (\vec{\nu} - \vec{\nu}_n)}{2}}
\]

\[
C_n = \begin{pmatrix}
\cos(\theta) & -\sin(\theta) \\
\sin(\theta) & \cos(\theta)
\end{pmatrix}
\begin{pmatrix}
\sigma_1^2 & 0 \\
0 & \sigma_2^2
\end{pmatrix}
\]
Spatio-temporal integration

How is probabilistic information pooled?

\[
\mathcal{N}(\vec{v}_n, C_n) = \frac{1}{\sqrt{\det(2\pi C_n)}} e^{-\frac{(\vec{v} - \vec{v}_n)^T C_n^{-1} (\vec{v} - \vec{v}_n)}{2}}
\]

\[
C_n = \begin{pmatrix}
\cos(\theta) & -\sin(\theta) \\
\sin(\theta) & \cos(\theta)
\end{pmatrix}
\begin{pmatrix}
\sigma_1^2 & 0 \\
0 & \sigma_2^2
\end{pmatrix}
\]

Independence of measurement noise

\[
P(\vec{v} | I) = \prod_n P(\vec{v} | I, n) = \mathcal{N}(\vec{v}_m, C)
\]

\[
\left\{
\begin{align*}
C^{-1} &= \sum C_n^{-1} \\
C^{-1} \cdot \vec{v}_m &= \sum C_n^{-1} \vec{v}_n
\end{align*}
\right.
\]
Spatio-temporal integration

Solution for the disk grating

\[ \gamma(C, d) = \frac{\frac{C^2}{C_e^2} \gamma_e(d)}{1 + \frac{C^2}{C_e^2} \gamma_e(d) + \frac{C^2}{C_i^2} \gamma_i(d)} \]

\[ \gamma_e(d) = 1 - \exp\left(-\frac{d^2}{2.\omega^2}\right) \]

\[ \gamma_i(d) = 1 - \exp\left(-\frac{d^2}{2.\omega_i^2}\right) \]
Generative model

Discrete formulation

\[
I(x + V(x, t) \cdot \delta t, t + \delta t) = I(x, t) + \mathcal{N}_I
\]
\[
V(x + V(x, t) \cdot \delta t, t + \delta t) = V(x, t) + \mathcal{N}_V
\]

Continuous formulation

\[
(V \cdot \nabla)I + \Delta_t I = \mathcal{N}_I
\]
\[
(V \cdot \nabla)V + \Delta_t V = \mathcal{N}_V
\]
Spontaneous state of the dynamical system
Fig. 1. Cartoon of the VI model, which represents an array of cortical units.

Fig. 11. Spatio-temporal subliminar influence of the